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New families of Super Mean Graphs

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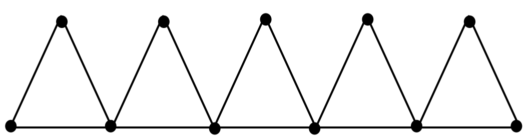
Abstract - Let G be a (p, q) graph and let $f : V(G) \rightarrow \{1, 2, ..., p+q\}$ be an injection. For each edge e = uv, let $f^*(e)=(f(u)+f(v))/2$ if f(u)+f(v) is even and $f^*(e) = ((f(u)+f(v))+1)/2$ if f(u)+f(v) is odd. Then f is called a super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, ..., p+q\}$. A graph that admits a super mean labeling is called a super mean graph.

In this paper, we prove that $C_n+v_1v_3$ (n≥4), Cube Q_3, Octahedron, the balloon of the triangular snake $T_n(C_m)$ n≥2, m≥3, m≠4, (2G, v_1, v_2) are super mean graphs.

Key words - labeling, mean labeling, super mean labeling, mean graph, super mean graph.

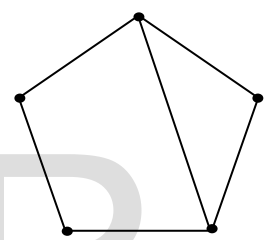
1 Introduction

Throughout this paper, by a graph we mean a finite, undirected, simple graph. Let G(V,E) be a graph with p vertices and q edges. For notations and terminology we follow [2]. Path on n vertices is denoted by Pn and a cycle on n vertices is denoted by C_n. A triangular snake T_n is obtained from a path $v_1, v_2, \ldots, v_n$ by joining v_i and v_{i+1} to a new vertex u_i for $1 \leq i \leq n-1$, that is, every edge of a path is replaced by a triangle C_3. For example, T_6 is shown in Figure 1.



T6
Figure 1

The graph Cn+v1v3 is obtained from the cycle Cn: $v_1v_2 \ldots v_nv_1$ by joining the vertices v_1 and v_3 by means of an edge. For example, $C_5+v_1v_3$ is shown in Figure 2.



$C_5+v_1v_3$
Figure 2

The graph $P_2 \times P_2 \times P_2$ is called the cube and is denoted by Q_3. Cube Q_3 is shown in Figure 3.

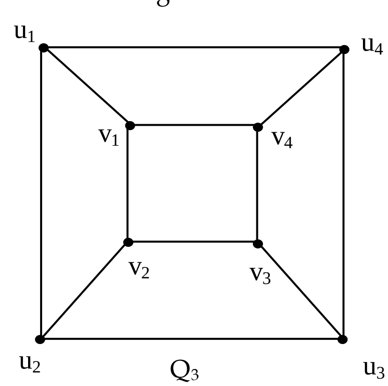


Figure 3

An octahedron is a polyhedron with 8 faces. An octahedron is shown in Figure 4.

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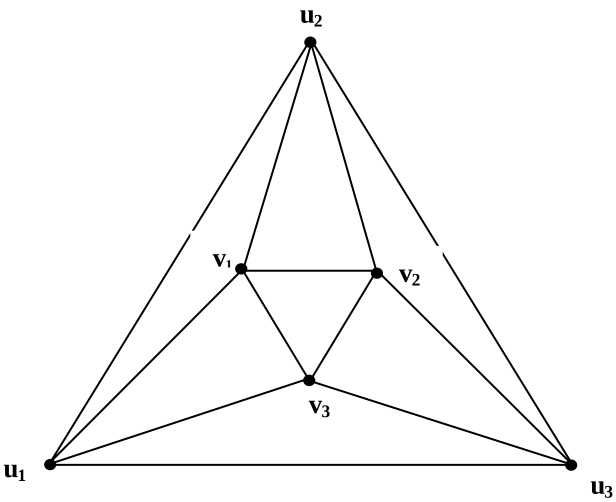


Figure 4

The balloon of the triangular snake $T_n(C_m)$ is the graph obtained from C_m by identifying an end vertex of the basic path in T_n at a vertex of C_m. For example, $T_5(C_6)$ is shown in Figure 5.

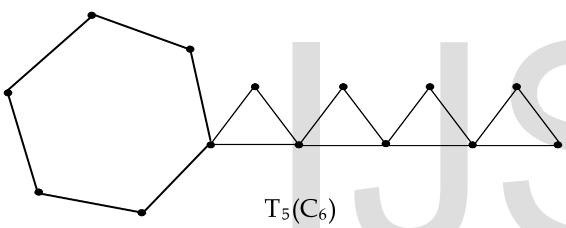
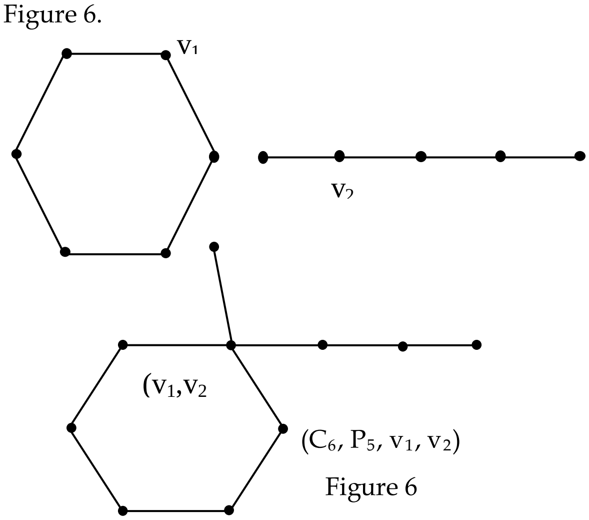

$T_5(C_6)$

Figure 5

Let G_1 and G_2 be two graphs with fixed vertices v_1 and v_2 respectively. Then (G_1, G_2, v_1, v_2) is the graph obtained from G_1 and G_2 by identifying the vertices v_1 and v_2. For example, the graph (C_6, P_5, v_1, v_2) is shown in Figure 6.



(C_6, P_5, v_1, v_2)

Figure 6

If $G_1 = G_2$, then (G, G, v_1, v_2) is denoted by $(2G, v_1, v_2)$. For example, $(2C_6, v_1, v_2)$ is shown in Figure 7.

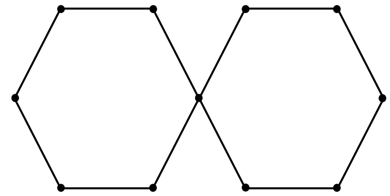

$(2C_6, v_1, v_2)$

Figure 7

A vertex labeling of G is an assignment $f : V(G) \rightarrow \{1, 2, 3, \ldots, p + q\}$ be an injection. For a vertex labeling f, an induced edge labeling f* is defined by

$$f^*(e) = \begin{cases} \frac{f(u) + f(v)}{2} & \text{if } f(u) \text{ and } f(v) \text{ are of same parity} \\ \frac{f(u) + f(v)}{2} & \text{otherwise} \end{cases}$$

A vertex labeling f is called a super mean labeling of G if its induced edge labeling $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \ldots, p + q\}$. If a graph has a super mean labeling, then we say that G is a super mean graph.

The concept of mean labeling was first introduced by S. Somasundaram and R. Ponraj [7]. They have studied in [5, 6, 7, 8] the mean labeling of some standard graphs.

The concept of super mean labeling was first introduced by R. Ponraj and D.Ramya [3]. They have studied [3, 4] the super meanness of some standard graphs like P_n, C_{2n+1}, $n \geq 1$, $K_n (n \leq 3)$, $K_{1,n}$ $(n \leq 3)$, T_n, $C_m \cup P_n (m\geq3, n\geq1)$, $B_{m,n}$ (m=n, n+1) etc. They have proved that the union of two super mean graphs is also a super mean graph and C4 is not a super mean graph. Also they determined all super mean graph of order ≤ 5. R. Vasuki and A. Nagarajan [10] proved that the super meanness of the graph C_{2n} for n≥3, the H-graph, carona of a H-graph, 2 – carona of a H-graph, carona of cycle C_n for n≥3, mC_n – snake for m≥1, n≥3 and $n \neq 4$ and $C_m \times P_n$ for m = 3, 5. In [1], the meanness of the following graphs have been proved: $C_m \times P_n$; the caterpillar P(n,2,3); $Q_3 \times P_{2n}$; carona of a H – graph; mC_3;

$C_n \cup K_{1,m}$ (n≥3, 1≤m≤4); $mC_3 \cup K_{1,m}$ (1≤m≤4); the dragon $P_n(C_m)$ and some standard graphs.

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In this paper, we prove the super meanness of the graph $C_n+v_1v_3$ ($n\geq4$), Cube Q_3, Octahedron, the balloon of the triangular snake $T_n(C_m)$ $n\geq2$, $m\geq3$, $m\neq4$, $(2G, v_1, v_2)$.

2 Super Mean Graphs

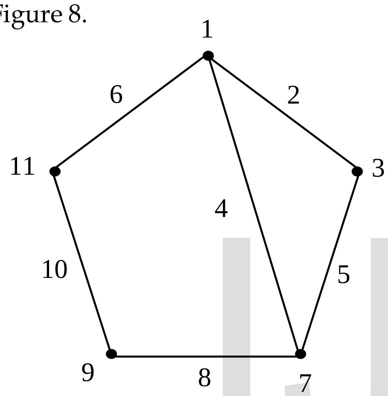
Theorem 2.1 $C_n+v_1v_3$ is a super mean graph for $n \geq 4$.

Proof Let C_n be a cycle with vertices $v_1, v_2, v_3, \ldots, v_n$ and edges $e_1, e_2, e_3, \ldots, e_n, e_{n+1}$.

Define $f : V(C_n+v_1v_3) \rightarrow \{1, 2, \ldots, n+1\}$ as follows:

Case 1 when n is odd, $n = 2m+1$, $m = 2, 3, 4, \ldots$

For $m = 2$, a super mean labeling of $C_5+v_1v_3$ is shown in Figure 8.



$C_5+v_1v_3$

Figure 8

For $m \geq 3$,

$f(v_i) = 2i-1$, $1\leq i \leq 2$; $f(v_{j+2}) = 2j+5$, $1 \leq j \leq 2$;

$f(v_{k+4}) = 4k+8$, $1 \leq k \leq m-2$;

$f(v_{m+l+2}) = 4m-4l+7$, $1 \leq l \leq m-2$;

$f(v_{2m+1}) = 10$.

Then the induced edge labels are

$f^*(e_1) = 2$; $f^*(e_i) = 3i-1$, $2 \leq i \leq 4$;

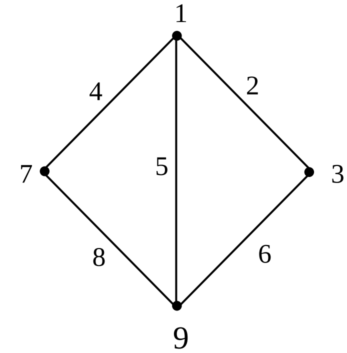
$f^*(e_{j+4}) = 2(2j+5)$, $1 \leq j \leq m-2$;

$f^*(e_{m+k+2}) = 4m-4k+5$, $1 \leq k \leq m-2$;

$f^*(e_{2m+1}) = 6$; $f^*(v_1v_3) = 4$.

Case 2 when n is even, $n = 2m$, $m = 2, 3, 4, \ldots$

For $m = 2$, a super mean labeling of $C_4+v_1v_3$ is shown in Figure 9.



$C_4+v_1v_3$

Figure 9

For $m \geq 3$,

$f(v_i) = 2i-1$, $1 \leq i \leq 2$; $f(v_3) = 7$;

$f(v_{j+3}) = 4j+5$, $1 \leq j \leq m-1$;

$f(v_{m+k+2}) = 4m-4k+2$, $1 \leq k \leq m-2$.

Then the induced edge labels are

$f^*(e_1) = 2$; $f^*(e_i) = 3i-1$, $2 \leq i \leq 3$;

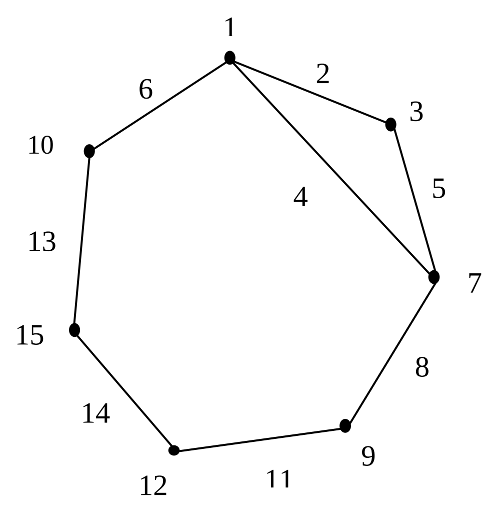
$f^*(e_{j+3}) = 4j+7$, $1 \leq j \leq m-2$;

$f^*(e_{m+k+1}) = 4m-4k+4$, $1 \leq k \leq m-2$;

$f^*(e_{2m}) = 6$; $f^*(v_1v_3) = 4$.

Clearly f is a super mean labeling of $C_n+v_1v_3$.

For example, the super mean labelings of $C_7+v_1v_3$ and $C_{10}+v_1v_3$ are shown in Figure 10. ■



(a) $C_7+v_1v_3$

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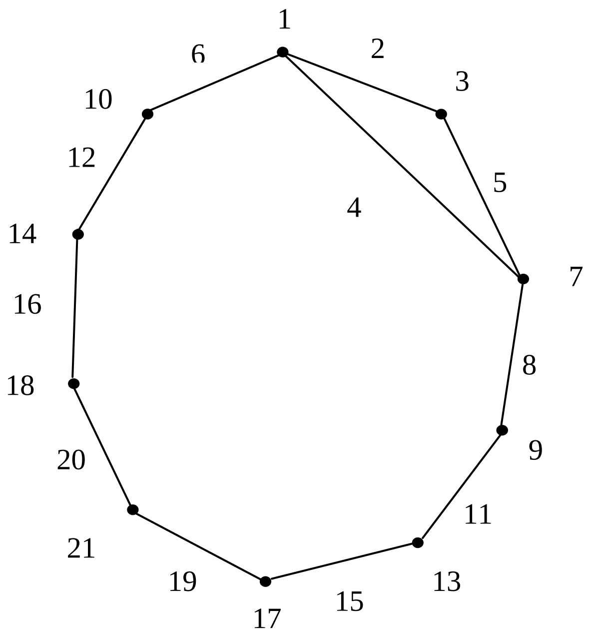


(b) $C_{10}+v_1v_3$

Figure 10

Theorem 2.2 Cube Q_3 is a super mean graph.

Proof Let u_1, u_2, u_3, u_4 and v_1, v_2, v_3, v_4 be the vertices of Q_3.

Define $f : V(Q_3) \to \{1, 2, 3, \dots, 20\}$ as follows:

$f(u_i) = 2i-1, 1 \le i \le 2$; $f(u_3) = 20$;

$f(u_4) = 18$; $f(v_1) = 11$;

$f(v_i) = 4i-3, 2 \le i \le 3$; $f(v_4) = 16$.

Then the induced edge labels are

$f^*(u_1u_2) = 2$; $f^*(u_iu_{i+1}) = 7i-2, 2 \le i \le 3$;

$f^*(u_1u_4) = 10$; $f^*(v_1v_2) = 8$;

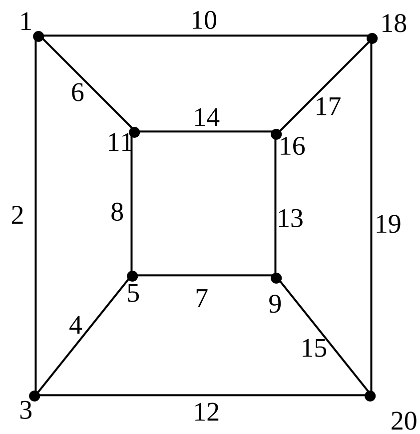
$f^*(v_iv_{i+1}) = 6i-5, 2 \le i \le 3$; $f^*(v_1v_4) = 14$;

$f^*(u_1v_1) = 6$; $f^*(u_2v_2) = 4$;

$f^*(u_3v_3) = 15$; $f^*(u_4v_4) = 17$.

Clearly f is a super mean labeling of Q_3.

For example, a super mean labeling of Q_3 is shown in Figure 11. ■



Q_3

Figure 11

Theorem 2.3 Octahedron is a super mean graph.

Proof Let u_1, u_2, u_3 and v_1, v_2, v_3 be the vertices of the octahedron.

Define $f : V(G) \to \{1, 2, 3, \dots, 18\}$ as follows:

$f(u_1) = 6$; $f(u_2) = 1$;

$f(u_3) = 13$; $f(v_1) = 3$;

$f(v_2) = 15$; $f(v_3) = 18$.

Then the induced edge labels are

$f^*(u_iu_{i+1}) = 3i+1, 1 \le i \le 2$; $f^*(u_1u_3) = 10$;

$f^*(v_iv_{i+1}) = 8i+1, 1 \le i \le 2$;

$f^*(v_1v_3) = 11$; $f^*(u_iv_i) = 3i+2, 1 \le i \le 2$;

$f^*(u_3v_3) = 16$; $f^*(v_1u_2) = 2$; $f^*(v_2u_3) = 14$;

$f^*(v_3u_1) = 12$.

Clearly f is a super mean labeling of the Octahedron.

For example, a super mean labeling of the Octahedron is shown in Figure 12. ■

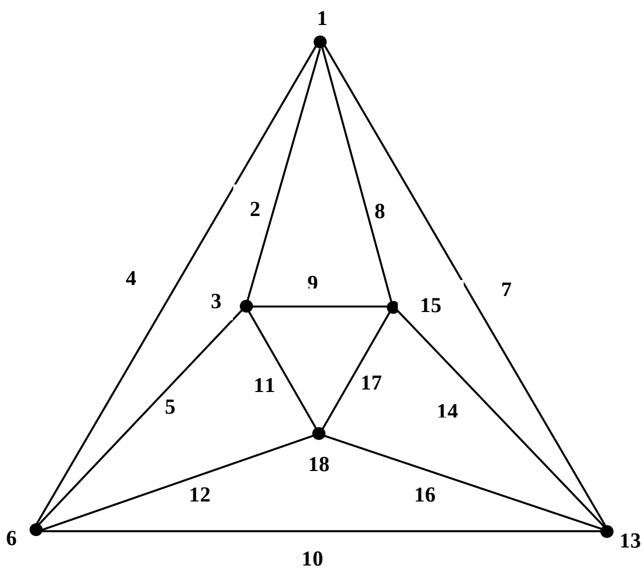


Figure 12

Theorem 2.4 $T_n(C_m)$ is a super mean graph for $n \geq 2$, $m \geq 3$, $m \neq 4$.

Proof Let $v_1, v_2, v_3, \ldots, v_m$ be the vertices of C_m and $u_1, u_2, u_3, \ldots, u_n$; $w_1, w_2, w_3, \ldots, w_{n-1}$ be the vertices of T_n.

Then define g on $T_n(C_m)$ as follows:

Case 1 when m is even, $m = 2k$, $k = 3, 4, 5, \ldots$

$g(v_i) = f(v_i), 1 \leq i \leq m$; $g(u_i) = 2m+5i-5, 1 \leq i \leq n$;

$g(w_i) = 2m+5i-3, 1 \leq i \leq n-1$.

Then the induced edge labels are

$g^*(e_i) = f(e_i), 1 \leq i \leq m$;

$g^*(u_iu_{i+1}) = 2m+5i-2, 1 \leq i \leq n-1$;

$g^*(u_iw=) = 2m+5i-4, 1 \leq i \leq n-1$;

$g^*(w_iu_{i+1}) = 2m+5i-1, 1 \leq i \leq n-1$.

Case 2 when m is odd, $m = 2k + 1$, $k = 2, 3, 4, \ldots$

$g(v_i) = f(v_i)$, $1 \leq i \leq m$; $g(u_i) = 2m+5i-5, 1 \leq i \leq n$;

$g(w_i) = 2m+5i-3, 1 \leq i \leq n-1$.

Then the induced edge labels are.

$g^*(e_i) = f(e_i), 1 \leq i \leq m$;

$g^*(u_iu_{i+1})= 2m+5i-2, 1 \leq i \leq n-1$;

$g^*(u_iw_i)= 2m+5i-4, 1 \leq i \leq n-1$;

$g^*(w_iu_{i+1}) = 2m+5i-1, 1 \leq i \leq n-1$.

Clearly g is a super mean labeling of $T_n(C_m)$.

For example, the super mean labelings of $T_5(C_6)$ and $T_5(C_9)$ are shown in Figure 13. ■

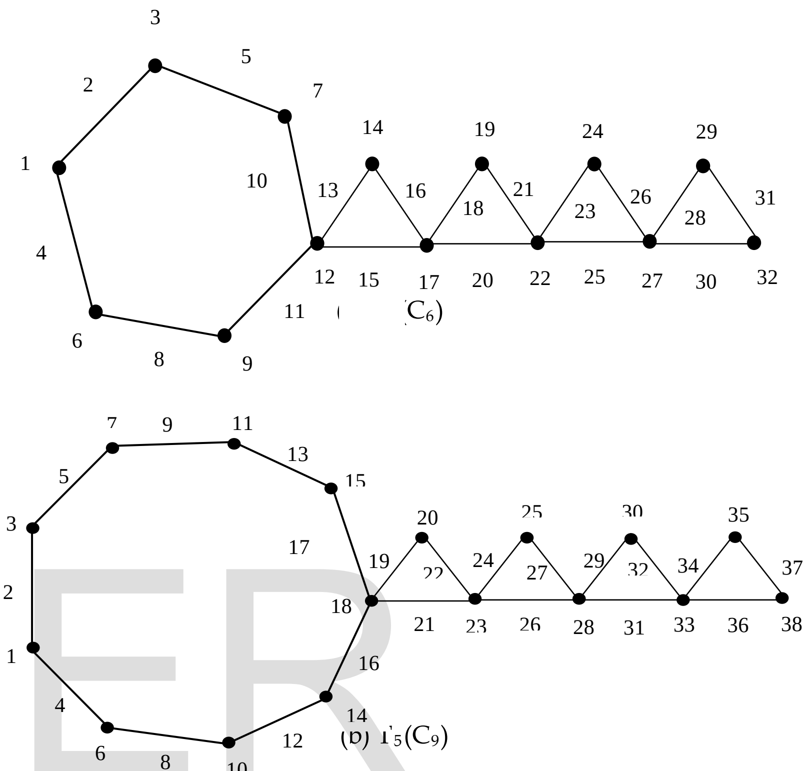


Figure 13

Theorem 2.5 If G is a super mean graph, then $(2G, v_1, v_2)$ is a super mean graph.

Proof Let u_1, u_2, u_3, u_4 and v_1, v_2, v_3, v_4 be the vertices of G and w_1, w_2, w_3, w_4 and x_1, x_2, x_3, x_4 be the vertices of $(2G, v_1, v_2)$.

Then define g on $(2G, v_1, v_2)$ as follows:

$g(u_i) = f(u_i), 1 \leq i \leq 4$; $g(v_i) = f(v_i), 1 \leq i \leq 4$;

$g(w_i) = f(u_i) + p+q-1, 1 \leq i \leq 4$;

$g(x_i) = f(v_i) +p+q-1, 1 \leq i \leq 4$.

Then the induced edge labels are

$g^*(e_i) = f(e_i), 1 \leq i \leq 4$; $g^*(e_j) = f(e_j), 1 \leq j \leq 4$;

$g^*(e_k) = f(e_k), 1 \leq k \leq 4$;

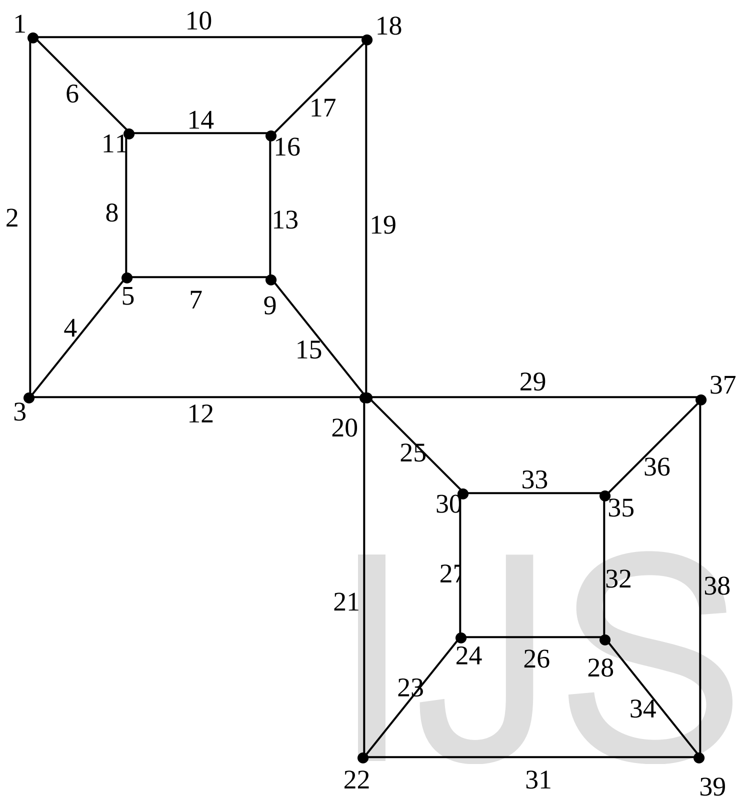
$g^*(e_l) = f(e_i) +p+q-1, 1 \leq i,l \leq 4$;

$g^*(y_i) = f(e_j) + p+q-1, 1 \le i,j \le 4;$

$g^*(z_i) = f(e_k)+p+q-1, 1 \le i,k \le 4.$

Clearly g is a super mean labeling of $(2G, v_1, v_2)$.

For example, a super mean labeling of $(2Q_3, v_1, v_2)$ is shown in Figure 14. ■



$(2Q_3, v_1, v_2)$

Figure 14

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